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MEMO

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SUBJECT: Errors in R10 - Latvel and Forvel Computations

Attached is a derivation of the exact computation of Latvel and Forvel, and a comparison of the approximation used in R10.

The conclusion drawn from this analysis is that the error in R10 does not lie in the resolution approximation.

Analysis of R10 Latvel and Forvel Computation - L. Berman

Current Algorithm

The algorithm used to compute VHL (Latvel) and VHF (Forvel) in R10 consists of two steps. First the moon-rate-corrected velocity, VMP, is resolved along horizontal axes parallel to and normal to the CSM plane:

$$\begin{aligned} VHY &= \overline{VMP} \cdot \overline{UHYP} \\ VHZ &= \overline{VMP} \cdot \overline{UHZP} \end{aligned} \tag{1}$$

where UHYP is normal to the CSM plane.

Next, using the assumption that the platform is aligned with its y-axis parallel to $\overline{UHYP}$, the velocity is transformed into body-axis related horizontal components by rotating thru the outer gimbal angle (o):

$$\begin{aligned} VHL &= VHZ \sin O + VHY \cos O \\ VHF &= VHZ \cos O - VHZ \sin O \end{aligned} \tag{2}$$

If, in fact, the platform is aligned with its y-axis along $\overline{UHYP}$,

$$\overline{UHYP} = (0,\ 1,\ 0) \tag{3}$$

If, in addition, the alignment is at the landing site, and we are near the landing site,

$$\overline{UHZP} = (0,\ 0,\ 1) \tag{4}$$

Then, from (1)

$$\begin{aligned} VHY &= VMPY \\ VHZ &= VMPZ, \end{aligned} \tag{5}$$

where we refer to the components in platform coordinates, and, finally,

$$\begin{aligned} VHL &= VMPY \cos O + VMPZ \sin O \\ VHF &= -VMPY \sin O + VMPZ \cos O \end{aligned} \tag{6}$$

Correct Computation

A correct computation depends on the definition of "forward" and "lateral". For example forward could be the intersection of the vehicle X-Z plane with the horizontal plane, or it could be the projection of the vehicle Z-axis on the horizontal. With both pitch and roll not zero, these two directions differ. Perhaps other definitions could be used. We will use the second definition in this analysis.

The simplest approach is to compute a body-related set of horizontal axes from the body axes, since these are available, and then resolve $\overline{VMP}$ into this system directly:

$$\begin{aligned} &\text{let } \overline{UY} = \overline{ZMB} \times \overline{UR} \\ &\text{and } \overline{UZ} = \overline{UR} \times \overline{UY} \end{aligned} \tag{7}$$

where UZ is now a horizontal projection of the body Z-axis, and UY is normal to it. Then

$$\begin{aligned} VHL &= \overline{VMP} \cdot \overline{UY} \\ VHF &= \overline{VMP} \cdot \overline{UZ} \end{aligned} \tag{8}$$

From pg. 5.6-43 of R-567 (Rev 7) we can deduce

$$\overline{ZNB} = \begin{Bmatrix} \sin O \sin M \cos I + \cos O \sin I \\ -\sin O \cos M \\ -\sin O \sin M \sin I + \cos O \cos I \end{Bmatrix} \tag{9}$$

If we again assume we are at the point of alignment:

$$\overline{UR} = (1,\ 0,\ 0), \tag{10}$$

$$\begin{aligned} &\text{then } \overline{UY} = (0,\ -\sin O \sin M \sin I + \cos O \cos I,\ \sin O \cos M) \\ &\text{and} \\ &\overline{UZ} = (0,\ -\sin O \cos M,\ -\sin O \sin M \sin I + \cos O \cos I) \end{aligned} \tag{11}$$

Finally, using (8), we get

$$VHL = VMPY\,(-\sin O \sin M \sin I + \cos O \cos I) + VMPZ(\sin O \cos M)$$

and (12)

$$VHF = VMPY\,(-\sin O \cos M) + VMPZ(-\sin O \sin M \sin I + \cos O \cos I)$$

If we assume I=M=O,

$$VHL = VMPY\,(\cos O) + VMPZ\,(\sin O)$$
$$VHF = VMPY\,(-\sin O) + VMPZ(\cos O) \qquad (13)$$

which is identical with (6).

<u>Error in Current Algorithm</u>

If we assume $I=M=5.7^{o}$

$$\cos I = \cos M = .995$$
$$\sin I = \sin M = .100$$

and from (12)

$$VHL = VMPY(-.01 \sin O + .995 \cos O) + VMPZ\,(.995 \sin O)$$
$$VHF = VMPY(-.995 \sin O) + VMPZ\,(-.01 \sin O + .995 \cos O) \qquad (14)$$

It can be seen that the error cannot get beyond about 1 or 2% of the velocity components. In fact, it is apparent that large inner and middle gimbal angles would be required to make the error significant, and even then it would be only a significant fraction of the actual components.

Thus we conclude that the errors seen in R10 do not arise from the analytical approximations used.